Lecture: 3-1 Derivatives of Polynomials and Exponentials

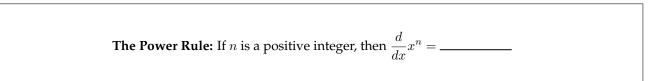
Derivative of a Constant Function: $\frac{d}{dx}(c) =$ ______

Example 1: Find the derivatives of the following functions.

(a) f(x) = 5.4 (b) $g(x) = \pi^7$ (c) $h(x) = \ln 2$

Example 2: Using the definition of the derivative, find the derivatives of the following functions.

(a) $f(x) = x^2$ (b) $f(x) = x^3$



Example 3: Find the derivatives of the following functions.

(a)
$$f(x) = x^9$$
 (b) $y = x^{99}$ (c) $\frac{d}{dt}(t^5)$

Using the definition of the derivative you can prove that the following derivatives. Does the power rule appear to hold for non-integer exponents as well?

(a)
$$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

(b) $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$

Example 4: Differentiate the following functions.

(a)
$$f(x) = \frac{1}{x^5}$$
 (b) $y = \sqrt[3]{x^5}$

Using the power rule we can find equations of tangent lines much more quickly! We can also find the **normal line**, which is defined as the line through a point *P* that is perpendicular to the tangent line at *P*.

Example 5: Find equations of the tangent line and normal line to the curve $y = x^2 \sqrt{x}$ at the point (1, 1).

The Constant Multiple Rule: If c is a constant and f is differentiable function then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x).$$

Example 6: Differentiate the following functions.

(a)
$$\frac{d}{dx}(5x^7)$$
 (b) $\frac{d}{dx}(-3\sqrt{x^5})$

The Sum/Difference Rule: If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x).$$

Example 7: Find the derivative of $y = x^7 + 10x^3 - 7x^2 + 2x - 9$.

Example 8: Find the points on the curve $y = x^4 - 2x^2 + 4$ where the tangent line is horizontal.

Example 9: Find the derivatives of the following functions.

(a)
$$y = (5x^2 - 2)^2$$
 (b) $f(x) = \frac{\sqrt{x + 2x - 3}}{x^3}$

Derivative of the Natural Exponential Function: $\frac{d}{dx}e^x = e^x$

Example 10: Find the derivatives of the following functions.

(a)
$$f(t) = \sqrt{3t} + \sqrt{\frac{3}{t}}$$
 (b) $f(x) = e^{x+2} + 4$

Example 11: At what point on the curve $y = e^x$ is the tangent line parallel to the line y - 5x = 2?

Example 13: Biologists have proposed a cubic function to model the length *L* of an Alaskan rockfish at age *A*:

$$L = 0.0155A^3 - 0.372A^2 + 3.95A + 1.21$$

where *L* is measured in inches and *A* in years. Calculate $\frac{dL}{dA}$ at A = 12 and interpret your answer.

Example 14: The equation of motion of a particle is $s = 2t^3 - 15t^2 + 36t + 1$. Find the velocity and acceleration functions. Then, determine the acceleration when the velocity is zero.

Example 15: Find the following limits.

(a)
$$\lim_{h \to 0} \frac{(2+h)^5 - 32}{h}$$
 (b) $\lim_{x \to 1} \frac{x^{99} - 1}{x - 1}$